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Extracting the jet azimuthal anisotropy from higher order cumulants

I.P. Lokhtin[a], L.I. Sarycheva[b], A.M. Snigirev[c]

M.V. Lomonosov Moscow State University, D.V. Skobeltsyn Institute of Nuclear Physics, 119992, Leninskie Gory, Moscow, Russia

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Abstract. We analyze the calculation method of the coefficient of the jet azimuthal anisotropy without reconstruction of the nuclear reaction plane, considering the higher order correlators between the azimuthal position of the jet axis and the angles of the particles not incorporated in the jet. The reliability of this technique in the real physical situation under LHC conditions is illustrated.

1 Introduction

Nowadays strong interest has arisen in the investigations and measurements of the azimuthal correlations in ultra-relativistic heavy ion collisions (see, for instance, [1] and references therein). One of the main reasons is that the rescattering and energy loss of hard partons in the azimuthally non-symmetric volume of dense quark–gluon matter, created initially in the nuclear overlap zone in collisions with non-zero impact parameter, can result in a visible azimuthal anisotropy of high-p_{T} hadrons at RHIC [1–4].

Recent anisotropic flow data at RHIC [5–7] can be described well by hydrodynamical models for semi-central collisions and p_{T} up to $\sim 2\,\mathrm{GeV}/c$ (the elliptic flow coefficient v_2 appears to be monotonously growing with increasing p_{T} [8] in this case), while the majority of microscopical Monte Carlo models underestimate the flow effects (see however [9]). The saturation and gradual decrease of v_2 at relatively large transverse momentum ($p_{\mathrm{T}} \gtrsim 2\,\mathrm{GeV}/c$), predicted as a signature of strong partonic energy loss in a dense QCD plasma, seem now to be supported by the preliminary data extending up to $p_{\mathrm{T}} \simeq 10\,\mathrm{GeV}/c$ at RHIC. The interpolation between the low-p_{T} relativistic hydrodynamics region and the high-p_{T} pQCD-computable region was evaluated in [4].

The initial gluon densities in Pb–Pb reactions at $\sqrt{s}_{NN} = 5.5\,\mathrm{TeV}$ at the Large Hadron Collider (LHC) are expected to be significantly higher than at RHIC, implying a stronger partonic energy loss. Moreover, since at LHC energies the inclusive cross section for hard jet production at $E_{\mathrm{T}} \sim 100\,\mathrm{GeV}$ is large enough to study the impact parameter dependence of such processes [10], one can hope to observe the azimuthal anisotropy for a hadronic jet itself [11,12]. In particular, the CMS experiment at LHC [13] will be able to provide both the jet reconstruction and an adequate measurement of the impact parameter of the nuclear collision using calorimetric information [14]. In the case of jets, the methodological advantage of azimuthal observables is that one needs to reconstruct only the azimuthal position of the jet without measuring its total energy. This can be done more easily and with a high accuracy, while the reconstruction of jet energy is a more ambiguous problem [14]. However the measurement of jet production as a function of the azimuthal angle requires event-by-event determination of the nuclear event plane based on the anisotropic flow analysis.

The methods for elliptic flow analysis can be generally divided in two categories: two-particle methods suggested and summarized in [15–17] and multi-particle methods [18,19]. In two-particle methods the contribution of non-flow (non-geometric) correlations to the determination of the elliptic flow coefficient v_2 is of the order of $1/\sqrt{N_0}$, where N_0 is the measured multiplicity. In multi-particle methods this contribution decreases typically as $1/N_0^{3/4}$, i.e., less by a factor of the order of $N_0^{1/4}$. Thus, experimental techniques based on a higher order cumulant analysis should be able in many cases to allow one to have access to the smaller values of the azimuthal particle anisotropy in comparison with two-particle methods, due to the automatic elimination of the major non-flow many-particle correlations and the systematic errors originating from the azimuthal asymmetry of the detector acceptance. This kind of analysis for particle flow has already been done by the STAR Collaboration at RHIC [20].

In our previous letter [21] we proposed the method for measurement of jet azimuthal anisotropy coefficients without direct reconstruction of the event plane and illustrated its reliability in a real experimental situation. This tech-

[a] e-mail: igor@lav01.sinp.msu.ru
[b] e-mail: lis@alex.sinp.msu.ru
[c] e-mail: snigirev@lav01.sinp.msu.ru

nique is based on the calculation of correlations between the azimuthal position of the jet axis and the angles of particles not incorporated in the jet, the azimuthal distribution of jets being described by the elliptic form. To improve our approach, in the present present paper we extend our analysis [21], considering the cumulant expansion [18] of multi-particle azimuthal correlations.

2 Correlators versus the jet elliptic anisotropy

Let us recall some features of our previous investigation [21]. We start from the essence of the techniques [15–17] for measuring the azimuthal elliptic anisotropy of the particle distribution, which can be written in the form

$$\frac{\mathrm{d}N}{\mathrm{d}\varphi} = \frac{N_0}{2\pi}\,[1+2v_2\cos 2(\varphi-\psi_{\mathrm{R}})],\quad N_0 = \int\limits_{-\pi}^{\pi} \mathrm{d}\varphi\,\frac{\mathrm{d}N}{\mathrm{d}\varphi}\,. \quad (1)$$

Knowing the nuclear reaction plane angle ψ_{R} allows one to determine the coefficient v_2 of the azimuthal anisotropy of the particle flow as an average (over particles) cosine of 2φ:

$$\langle\cos 2(\varphi-\psi_{\mathrm{R}})\rangle = \frac{1}{N_0}\int\limits_{-\pi}^{\pi}\mathrm{d}\varphi\ \cos 2(\varphi-\psi_{\mathrm{R}})\ \frac{\mathrm{d}N}{\mathrm{d}\varphi} = v_2. \quad (2)$$

However in the case when there are no other correlations of particles except those due to flow (or such other correlations can be neglected), the coefficient of azimuthal anisotropy can be determined using a two-particle azimuthal correlator without the event plane angle ψ_{R}:

$$\begin{aligned}&\langle\cos 2(\varphi_1-\varphi_2)\rangle\\ &= \frac{1}{N_0^2}\int\limits_{-\pi}^{\pi}\mathrm{d}\varphi_1\int\limits_{-\pi}^{\pi}\mathrm{d}\varphi_2\ \cos 2(\varphi_1-\varphi_2)\ \frac{\mathrm{d}^2N}{\mathrm{d}\varphi_1\mathrm{d}\varphi_2}\\ &= \frac{1}{N_0^2}\int\limits_{-\pi}^{\pi}\mathrm{d}\varphi_1\int\limits_{-\pi}^{\pi}\mathrm{d}\varphi_2\ \cos 2((\varphi_1-\psi_{\mathrm{R}})-(\varphi_2-\psi_{\mathrm{R}}))\\ &\quad\times\frac{\mathrm{d}N}{\mathrm{d}\varphi_1}\ \frac{\mathrm{d}N}{\mathrm{d}\varphi_2} = v_2^2. \end{aligned}\quad (3)$$

Let us consider now the event with high-p_{T} jet (dijet) production, the distribution of jets over the azimuthal angle relative to the reaction plane being described well by the elliptic form [11],

$$\begin{aligned}\frac{\mathrm{d}N^{\mathrm{jet}}}{\mathrm{d}\varphi} &= \frac{N_0^{\mathrm{jet}}}{2\pi}\,[1+2v_2^{\mathrm{jet}}\cos 2(\varphi-\psi_{\mathrm{R}})],\\ N_0^{\mathrm{jet}} &= \int\limits_{-\pi}^{\pi}\mathrm{d}\varphi\,\frac{\mathrm{d}N^{\mathrm{jet}}}{\mathrm{d}\varphi},\end{aligned}\quad (4)$$

where the coefficient of the jet azimuthal anisotropy v_2^{jet} is determined as an average over all events cosines of 2φ,

$$\begin{aligned}&\langle\cos 2(\varphi-\psi_{\mathrm{R}})\rangle_{\mathrm{event}}\\ &= \frac{1}{N_0^{\mathrm{jet}}}\int\limits_{-\pi}^{\pi}\mathrm{d}\varphi\ \cos 2(\varphi-\psi_{\mathrm{R}})\ \frac{\mathrm{d}N^{\mathrm{jet}}}{\mathrm{d}\varphi} = v_2^{\mathrm{jet}}.\end{aligned}\quad (5)$$

One can calculate the correlator between the azimuthal position of the jet axis φ_{jet} [1] and the angles of particles which are not incorporated in the jet(s). The value of this correlator is related to the elliptic coefficients v_2 and v_2^{jet} by

$$\begin{aligned}&\langle\langle\cos 2(\varphi_{\mathrm{jet}}-\varphi)\rangle\rangle_{\mathrm{event}}\\ &= \frac{1}{N_0^{\mathrm{jet}}N_0}\int\limits_{-\pi}^{\pi}\mathrm{d}\varphi_{\mathrm{jet}}\int\limits_{-\pi}^{\pi}\mathrm{d}\varphi\ \cos 2(\varphi_{\mathrm{jet}}-\varphi)\ \frac{\mathrm{d}N^{\mathrm{jet}}}{\mathrm{d}\varphi_{\mathrm{jet}}}\ \frac{\mathrm{d}N}{\mathrm{d}\varphi}\\ &= \frac{1}{N_0^{\mathrm{jet}}}\int\limits_{-\pi}^{\pi}\mathrm{d}\varphi_{\mathrm{jet}}\ \cos 2(\varphi_{\mathrm{jet}}-\psi_{\mathrm{R}})\ \frac{\mathrm{d}N^{\mathrm{jet}}}{\mathrm{d}\varphi_{\mathrm{jet}}}\ v_2 = v_2^{\mathrm{jet}}\ v_2.\end{aligned}\quad (6)$$

Using (3) and the intermediate result in (6) (after averaging over particles $\cos 2(\varphi_{\mathrm{jet}}-\varphi)$ reduces to $v_2\cos 2(\varphi_{\mathrm{jet}}-\psi_{\mathrm{R}})$) we derive the formula for computing the absolute value of the coefficient of the jet azimuthal anisotropy (without reconstruction of the sign of v_2^{jet}):

$$v_2^{\mathrm{jet}} = \left\langle\frac{\langle\cos 2(\varphi_{\mathrm{jet}}-\varphi)\rangle}{\sqrt{\langle\cos 2(\varphi_1-\varphi_2)\rangle}}\right\rangle_{\mathrm{event}}. \quad (7)$$

This formula does not require the direct determination of the reaction plane angle ψ_{R}. The brackets $\langle\ \rangle$ represent the averaging over particles (not incorporated in the jet) in a given event, while the brackets $\langle\ \rangle_{\mathrm{event}}$ are the averaging over events.

The formula (7) can be generalized by introducing as weights the particle momenta,

$$v_{2(p)}^{\mathrm{jet}} = \left\langle\frac{\langle\, p_{\mathrm{T}}(\varphi)\cos 2(\varphi_{\mathrm{jet}}-\varphi)\rangle}{\sqrt{\langle\, p_{\mathrm{T}1}(\varphi_1)\ p_{\mathrm{T}2}(\varphi_2)\ \cos 2(\varphi_1-\varphi_2)\rangle}}\right\rangle_{\mathrm{event}}. \quad (8)$$

In this case the brackets $\langle\ \rangle$ denote the averaging over angles and transverse momenta of the particles. The other modification of (8),

$$v_{2(E)}^{\mathrm{jet}} = \left\langle\frac{\langle\, E(\varphi)\cos 2(\varphi_{\mathrm{jet}}-\varphi)\rangle}{\sqrt{\langle\, E_1(\varphi_1)\ E_2(\varphi_2)\cos 2(\varphi_1-\varphi_2)\rangle}}\right\rangle_{\mathrm{event}} \quad (9)$$

($E_i(\varphi_i)$ being the energy deposit in a calorimetric segment i of position φ_i), allows one to use the calorimetric measurements (9) for the determination of the jet azimuthal anisotropy.

[1] The other possibility is to fix the azimuthal position of a leading particle in the jet. In this case calculating azimuthal correlations can provide the information on the azimuthal anisotropy of the high-p_{T} particle spectrum

3 Higher order correlators

The main advantage of the higher order cumulant analysis lies in the fact that, as argued in [18], if the flow is larger than the non-flow correlations, the contribution of the latter to v_2 extracted from higher order correlators is suppressed[2] by powers of the particle multiplicity N_0 in an event.

Thus, for example, the fourth order cumulant for elliptic particle flow is defined as [18]

$$\begin{aligned} c_2[4] \equiv & \langle \cos 2(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4) \rangle \\ & - \langle \cos 2(\varphi_1 - \varphi_3) \rangle \, \langle \cos 2(\varphi_2 - \varphi_4) \rangle \\ & - \langle \cos 2(\varphi_1 - \varphi_4) \rangle \, \langle \cos 2(\varphi_2 - \varphi_3) \rangle, \end{aligned} \quad (10)$$

and in the case of there existing only correlations with the reaction plane (i.e. the factorization of multi-particle distributions is held as in (3)) is equal to

$$c_2[4] \; = \; -v_2^4. \quad (11)$$

If now one defines the coefficient v_2 of the azimuthal anisotropy through the two-particle correlator

$$v_2 \; = \; \sqrt{\langle \cos 2(\varphi_1 - \varphi_2) \rangle}, \quad (12)$$

then the contribution of non-flow correlations, as argued in [18], is of order $1/\sqrt{N_0}$, while their contribution to v_2, extracted from the fourth order correlator

$$v_2 \; = \; (-c_2[4])^{1/4}, \quad (13)$$

scales as $1/N_0^{3/4}$, i.e. it is suppressed by an extra factor of $1/N_0^{1/4}$. The corresponding data analysis based on (10) with the result (11) has been already carried out at STAR [20].

Now using the derivation of (7) and the result (11) it is straightforward to obtain the formula for the calculation (measurement) of the coefficient of the jet azimuthal anisotropy through the higher order correlator, which is less sensitive to non-flow correlations:

$$\begin{aligned} v_2^{\mathrm{jet}}[4] \; = & \Big\langle \frac{1}{(-c_2[4])^{3/4}} \\ & \times [-\langle \cos 2(\varphi_{\mathrm{jet}} + \varphi_1 - \varphi_2 - \varphi_3) \rangle \\ & + \langle \cos 2(\varphi_{\mathrm{jet}} - \varphi_2) \rangle \;\; \langle \cos 2(\varphi_1 - \varphi_3) \rangle \\ & + \langle \cos 2(\varphi_{\mathrm{jet}} - \varphi_3) \rangle \; \langle \cos 2(\varphi_1 - \varphi_2) \rangle] \Big\rangle_{\mathrm{event}} . \end{aligned} \quad (14)$$

Here we stress once more that in the case of there existing only correlations with the reaction plane, (14) together with (7) transforms into an identity. The formula just derived can be, as in Sect. 2, generalized for calorimetric measurements of energy flows:

$$\begin{aligned} v_{2(E)}^{\mathrm{jet}}[4] \; = & \Big\langle \frac{1}{(-c_{2(E)}[4])^{3/4}} \\ & \times [-\langle E_1(\varphi_1) E_2(\varphi_2) E_3(\varphi_3) \cos 2(\varphi_{\mathrm{jet}} + \varphi_1 - \varphi_2 - \varphi_3) \rangle \\ & + \langle E_2(\varphi_2) \cos 2(\varphi_{\mathrm{jet}} - \varphi_2) \rangle \\ & \quad \times \langle E_1(\varphi_1) E_3(\varphi_3) \cos 2(\varphi_1 - \varphi_3) \rangle \\ & + \langle E_3(\varphi_3) \cos 2(\varphi_{\mathrm{jet}} - \varphi_3) \rangle \\ & \quad \times \langle E_1(\varphi_1) E_2(\varphi_2) \cos 2(\varphi_1 - \varphi_2) \rangle] \Big\rangle_{\mathrm{event}}, \end{aligned} \quad (15)$$

where

$$\begin{aligned} & c_{2(E)}[4] \\ & = \langle E_1(\varphi_1) E_2(\varphi_2) E_3(\varphi_3) E_4(\varphi_4) \\ & \qquad \times \cos 2(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4) \rangle \\ & - \langle E_1(\varphi_1) E_3(\varphi_3) \cos 2(\varphi_1 - \varphi_3) \rangle \\ & \qquad \times \langle E_2(\varphi_2) E_4(\varphi_4) \cos 2(\varphi_2 - \varphi_4) \rangle \\ & - \langle E_1(\varphi_1) E_4(\varphi_4) \cos 2(\varphi_1 - \varphi_4) \rangle \\ & \qquad \times \langle E_2(\varphi_2) E_3(\varphi_3) \cos 2(\varphi_2 - \varphi_3) \rangle. \end{aligned} \quad (16)$$

In the case when the azimuthal position of the jet axis correlates not only with the reaction plane, one can try to improve this technique using the multiple correlators of another form: not averaging over all events but selecting some of their sub-events. For instance, one can consider sub-events 1 and 2, when jets are produced with the rapidity $y > 0$ and $y < 0$. Then calculating the correlator

$$\begin{aligned} & c_2^{\mathrm{jet}}[4] \\ & = \Big\langle \frac{1}{\sqrt{\langle \cos 2(\varphi_1 - \varphi_2) \rangle \langle \cos 2(\phi_1 - \phi_2) \rangle}} \\ & \quad \times [\langle \cos 2(\varphi_{\mathrm{jet}} - \varphi + \phi_{\mathrm{jet}} - \phi) \rangle \\ & \quad + \langle \cos 2(\varphi_{\mathrm{jet}} - \varphi - \phi_{\mathrm{jet}} + \phi) \rangle \\ & \quad - \langle \cos 2(\varphi_{\mathrm{jet}} - \varphi) \rangle \; \langle \cos 2(\phi_{\mathrm{jet}} - \phi) \rangle] \Big\rangle_{\mathrm{sub-event\ 1,\ 2}}, \end{aligned} \quad (17)$$

we obtain that, if there are flow particle correlations only and the distribution of the jets over the azimuthal angles is described by the elliptic form (4) in every sub-event, it is equal to

$$c_2^{\mathrm{jet}}[4] \; = \; v_2^{\mathrm{jet}}(y > 0) \; v_2^{\mathrm{jet}}(y < 0). \quad (18)$$

In (17) the angles φ are defined as the azimuthal angles of particles and jets in a sub-event with $y > 0$, and ϕ in a sub-event with $y < 0$. Correspondingly the brackets $\langle \;\; \rangle$ represent the averaging over particles in sub-events 1, 2, while the brackets $\langle \;\; \rangle_{\mathrm{sub-event\ 1,\ 2}}$ are the averaging over these sub-events. The generalization of (17) for calorimetric measurements of the energy flow is obvious (similar to (9) and (15)). We do not also write this result specially as examples of utilizing sixth and other higher order correlators.

[2] This can be essential under data analysis with a not high enough multiplicity of particles in an event

4 Non-flow correlations

Here we discuss the influence of non-flow correlations[3] on the $v_2^{\rm jet}$ determination. There are various sources of such correlations, among which we have minijet production [22], global momentum conservation [23,24], resonance decays (in which the decay products are correlated), final state Coulomb, and strong or quantum interactions [25]. We restrict our consideration to two-particle correlations only. This will be enough to illustrate the advantage in using higher order cumulants. In this case multi-particle distributions are not factorized again and instead of (3) we have

$$\begin{aligned} c_2[2] \;&\equiv\; \langle \cos 2(\varphi_1-\varphi_2)\rangle \\ &= \frac{1}{N_2}\int\limits_{-\pi}^{\pi}{\rm d}\varphi_1\int\limits_{-\pi}^{\pi}{\rm d}\varphi_2\;\cos 2(\varphi_1-\varphi_2) \\ &\quad\times\left[\frac{{\rm d}N}{{\rm d}\varphi_1}\frac{{\rm d}N}{{\rm d}\varphi_2}+\frac{{\rm d}N_{\rm cor}}{{\rm d}\varphi_1{\rm d}\varphi_2}\right] \\ &= v_2^2\,\frac{1+v_{\rm cor}^{-}/v_2^2}{1+\varDelta}, \end{aligned} \tag{19}$$

where

$$\begin{aligned} N_2 &= \int\limits_{-\pi}^{\pi}{\rm d}\varphi_1\int\limits_{-\pi}^{\pi}{\rm d}\varphi_2\left[\frac{{\rm d}N}{{\rm d}\varphi_1}\frac{{\rm d}N}{{\rm d}\varphi_2}+\frac{{\rm d}N_{\rm cor}}{{\rm d}\varphi_1{\rm d}\varphi_2}\right], \\ \varDelta &= \frac{1}{N_0^2}\int\limits_{-\pi}^{\pi}{\rm d}\varphi_1\int\limits_{-\pi}^{\pi}{\rm d}\varphi_2\,\frac{{\rm d}N_{\rm cor}}{{\rm d}\varphi_1{\rm d}\varphi_2}, \qquad (20) \\ v_{\rm cor}^{-} &= \frac{1}{N_0^2}\int\limits_{-\pi}^{\pi}{\rm d}\varphi_1\int\limits_{-\pi}^{\pi}{\rm d}\varphi_2\;\cos 2(\varphi_1-\varphi_2)\;\frac{{\rm d}N_{\rm cor}}{{\rm d}\varphi_1{\rm d}\varphi_2}. \end{aligned}$$

Equation (6) remains unchanged and the result (7) transforms into

$$\begin{aligned} v_2^{\rm jet}[2] &\equiv \left\langle \frac{\langle\cos 2(\varphi_{\rm jet}-\varphi)\rangle}{\sqrt{\langle\cos 2(\varphi_1-\varphi_2)\rangle}}\right\rangle_{\rm event} \\ &= v_2^{\rm jet}\sqrt{\frac{1+\varDelta}{1+v_{\rm cor}^{-}/v_2^2}}. \end{aligned} \tag{21}$$

After some algebra the fourth order cumulant (10) reduces to

$$\begin{aligned} c_2[4] \;&=\; v_2^4 \\ &\times\frac{1+4v_{\rm cor}^{-}/v_2^2+2v_{\rm cor}^{--}/v_2^4+2v_{\rm cor}^{+}/v_2^2+v_{\rm cor}^{++}/v_2^4}{1+6\varDelta+3\varDelta^2} \\ &-2v_2^4\left(\frac{1+v_{\rm cor}^{-}/v_2^2}{1+\varDelta}\right)^2, \end{aligned} \tag{22}$$

where

$$\begin{aligned} v_{\rm cor}^{+} &= \frac{1}{N_0^2}\int\limits_{-\pi}^{\pi}{\rm d}\varphi_1\int\limits_{-\pi}^{\pi}{\rm d}\varphi_2\;\cos 2(\varphi_1-\psi_{\rm R}+\varphi_2-\psi_{\rm R}) \\ &\quad\times\frac{{\rm d}N_{\rm cor}}{{\rm d}\varphi_1{\rm d}\varphi_2}, \\ v_{\rm cor}^{++} &= \frac{1}{N_0^4}\int\limits_{-\pi}^{\pi}{\rm d}\varphi_1\int\limits_{-\pi}^{\pi}{\rm d}\varphi_2\int\limits_{-\pi}^{\pi}{\rm d}\varphi_3\int\limits_{-\pi}^{\pi}{\rm d}\varphi_4 \\ &\quad\times\cos 2(\varphi_1+\varphi_2-\varphi_3-\varphi_4)\,\frac{{\rm d}N_{\rm cor}}{{\rm d}\varphi_1{\rm d}\varphi_2}\,\frac{{\rm d}N_{\rm cor}}{{\rm d}\varphi_3{\rm d}\varphi_4}, \\ v_{\rm cor}^{--} &= \frac{1}{N_0^4}\int\limits_{-\pi}^{\pi}{\rm d}\varphi_1\int\limits_{-\pi}^{\pi}{\rm d}\varphi_2\int\limits_{-\pi}^{\pi}{\rm d}\varphi_3\int\limits_{-\pi}^{\pi}{\rm d}\varphi_4 \\ &\quad\times\cos 2(\varphi_1+\varphi_2-\varphi_3-\varphi_4)\,\frac{{\rm d}N_{\rm cor}}{{\rm d}\varphi_1{\rm d}\varphi_3}\,\frac{{\rm d}N_{\rm cor}}{{\rm d}\varphi_2{\rm d}\varphi_4} \\ &= (v_{\rm cor}^{-})^2-\left(\frac{1}{N_0^2}\int\limits_{-\pi}^{\pi}{\rm d}\varphi_1\int\limits_{-\pi}^{\pi}{\rm d}\varphi_3\;\sin 2(\varphi_1-\varphi_3)\right. \\ &\quad\left.\times\frac{{\rm d}N_{\rm cor}}{{\rm d}\varphi_1{\rm d}\varphi_3}\right)^2. \end{aligned} \tag{23}$$

This is $(v_{\rm cor}^{-})^2$ if $\frac{{\rm d}N_{\rm cor}}{{\rm d}\varphi_1{\rm d}\varphi_2}$ is an even function of the angular difference $(\varphi_1-\varphi_2)$.

The numerator in (14) is rewritten in the following form:

$$\begin{aligned} {\rm Num} &\equiv -\langle\cos 2(\varphi_{\rm jet}+\varphi_1-\varphi_2-\varphi_3)\rangle \\ &\quad+\langle\cos 2(\varphi_{\rm jet}-\varphi_2)\rangle\langle\cos 2(\varphi_1-\varphi_3)\rangle \\ &\quad+\langle\cos 2(\varphi_{\rm jet}-\varphi_3)\rangle\langle\cos 2(\varphi_1-\varphi_2)\rangle \\ &= -\;v_2^3\cos 2(\varphi_{\rm jet}-\psi_{\rm R})\;\frac{1+2v_{\rm cor}^{-}/v_2^2+v_{\rm cor}^{+}/v_2^2}{1+3\varDelta} \\ &\quad+2v_2^3\;\cos 2(\varphi_{\rm jet}-\psi_{\rm R})\;\frac{1+v_{\rm cor}^{-}/v_2^2}{1+\varDelta}+{\rm SIN}, \end{aligned} \tag{24}$$

where the terms SIN are proportional to $\sin 2(\varphi_{\rm jet}-\psi_{\rm R})$ and vanishing after averaging over $\varphi_{\rm jet}$.

At first glance it is hard to see the advantage in using higher order cumulants from (21), (22) and (24). However, it is reasonable to suppose that the contribution of two-particle correlations to the normalization factor N_2 is small, $\varDelta \ll 1$, while their "second Fourier harmonic" $v_{\rm cor}^{-}$ can be of the order of v_2^2. Then *all* direct two-particle correlations $v_{\rm cor}^{-}$ are automatically *canceled out*[4] from (22) and (24) in first order in $\varDelta$, but *survive* in (21). The non-direct two-particle correlations $v_{\rm cor}^{+}$, $v_{\rm cor}^{++}$ survive. But they are suppressed in comparison with the direct correlations $v_{\rm cor}^{-}$ (contributing to $v_2^{\rm jet}[2]$) due to the fact that $\frac{{\rm d}N_{\rm cor}}{{\rm d}\varphi_1{\rm d}\varphi_2}$ is an even function of the angular difference $(\varphi_1-\varphi_2)$ only, in most physically interesting cases [22]. Moreover, for the small-angle δ-like correlations $\left(\frac{{\rm d}N_{\rm cor}}{{\rm d}\varphi_1{\rm d}\varphi_2}\sim\frac{\exp(-(\varphi_1-\varphi_2)^2/2\sigma^2)}{\sqrt{2\pi}\sigma},\sigma\to 0\right)$ and for large-angle oscillating ones $\left(\frac{{\rm d}N_{\rm cor}}{{\rm d}\varphi_1{\rm d}\varphi_2}\sim\cos 2(\varphi_1-\varphi_2)\right)$ the non-direct correlations $v_{\rm cor}^{+}$, $v_{\rm cor}^{++}$ are equal to zero. Then

[3] See also the appendix of [18] and [22]

[4] This is one of the main advantage of the cumulant expansion

$$v_2^{\text{jet}}[4] = \left\langle \frac{\text{Num}}{(-c_2[4])^{3/4}} \right\rangle_{\text{event}} \simeq v_2^{\text{jet}} \qquad (25)$$

in this case, while

$$v_2^{\text{jet}}[2] \simeq v_2^{\text{jet}} \sqrt{\frac{1}{1 + v_{\text{cor}}^{-}/v_2^2}}. \qquad (26)$$

Thus (25) and (26) demonstrate the better accuracy of the higher order cumulants explicitly.

5 Discussion

In order to illustrate the applicability of the method presented with regard for the real physical situation, we consider the following model (see [21] for details).

5.1 The model

The initial jet distributions in a nucleon–nucleon subcollision at $\sqrt{s} = 5.5\,\text{TeV}$ have been generated using PYTHIA_5.7 [26]. We simulated the rescattering and energy loss of jets in a gluon-dominated plasma, created initially in the nuclear overlap zone in Pb–Pb collisions at different impact parameters. For details of this approach one can refer to [10,11]. Essential for our consideration here is that in non-central collisions the azimuthal distribution of the jets is approximated well by the elliptic form (4). In the model the coefficient of the jet azimuthal anisotropy increases almost linearly with the growth of the impact parameter b and becomes a maximum at $b \sim 1.2R_A$, where R_A is the nucleus radius. After that, v_2^{jet} drops rapidly with increasing b: this is the domain of impact parameter values where the effect of decreasing energy loss due to a reducing effective transverse size of the dense zone and initial energy density of the medium is crucial and not compensated anymore by the stronger non-symmetry of the volume. The following kinematical cuts on the jet transverse energy and rapidity have been applied: $E_{\text{T}}^{\text{jet}} > 100\,\text{GeV}$ and $|y^{\text{jet}}| < 1.5$. After this the dijet event is superimposed on the Pb–Pb event containing an anisotropic flow.

Anisotropic flow was generated using the simple hydrodynamical Monte Carlo code [27,21] giving a hadron (charged and neutral pion, kaon and proton) spectrum as a superposition of the thermal distribution and collective flow. To be definite, we fixed the following "freeze-out" parameters: the temperature $T_{\text{f}} = 140\,\text{MeV}$, the collective longitudinal rapidity $Y_{\text{L}}^{\max} = 3$ and the collective transverse rapidity $Y_{\text{T}}^{\max} = 1$. We set the Poisson multiplicity distribution and took into account the impact parameter dependence of the multiplicity in a simple way, just suggesting that the mean multiplicity of particles is proportional to the nuclear overlap function. We also suggested [21] that the spatial ellipticity of the "freeze-out" region is directly related to the initial spatial ellipticity of the nuclear overlap zone. Such "scaling" allows one to avoid using additional parameters and, at the same time, results in an elliptic anisotropy of particle and energy flow due to the dependence of the effective transverse size of the "freeze-out" region on the azimuthal angle of a "hadronic liquid" element. Obtained in such a way the azimuthal distribution of particles is described well by the elliptic form (1) for the domain of reasonable impact parameter values.

To be specific, we consider the geometry of the CMS detector [13] at LHC. The central ("barrel") part of the CMS calorimetric system covers the pseudo-rapidity region $|\eta| < 1.5$, the segmentation of electromagnetic and hadron calorimeters being $\Delta\eta \times \Delta\phi = 0.0174 \times 0.0174$ and $\Delta\eta \times \Delta\phi = 0.0872 \times 0.0872$ respectively [13]. In order to reproduce roughly the experimental conditions (not including real detector effects, but just assuming calorimeter hermeticity), we applied (9) and (15) to the energy deposition $E_i(\varphi_i)$ of the generated particles, integrated over the rapidity in 72 segments (according to the number of segments in the hadron calorimeter: $72 \times 0.0872 = 2\pi$; $i = 1, ..., 72$) covering the full azimuth.

Note that in the CMS heavy ion physics program, the modified sliding window-type jet finding algorithm has been developed to search for "jet-like" clusters above the average energy, and to subtract the background from the underlying event [14]. Strictly speaking, after jet extraction the background energy deposition in the calorimetric cells should be redefined and can appear to be not exactly equal to the initially generated one. However we neglect this effect here. In a real experimental situation, in order to avoid the influence of the jet contribution on the particle flow, one can consider jets and particles incorporated in the energy flow analysis in different rapidity regions.

5.2 Numerical results

We have found [21] that the accuracy of the v_2^{jet} determination from (9) is close to 100% for a semi-central ($b \lesssim R_A$) collision and becomes significantly worse in a very peripheral collision ($b \sim 2R_A$), wherein decreasing multiplicity and azimuthal anisotropy of the event results in relatively large fluctuations of the energy deposition in each segment.

In the present paper we tested the efficiency of the higher order correlator (15) and have found, at first, that the results for v_2^{jet} obtained from (9) and (15) are practically the same. This is explained by the fact that our simple Monte Carlo event generator gives the elliptic anisotropy of the energy flow, correlated with the reaction plane, but no correlations between the energy depositions in the calorimeter segments. We can introduce such correlations at the calorimetric level "by hand", simply assuming that the probability of finding the energy E_i in a segment i and the energy E_j in a segment j is proportional to $E_iE_j(1 + c_{ij})$, where the "correlation strength" c_{ij} may be, for example, proportional to δ_{ij} (the short-range δ-like correlations) or $\cos 2(\varphi_i - \varphi_j)$ (the long-range oscillating correlations). In this case we became convinced that the higher order cumulant (15) was almost independent of such correlations (as it was shown in Sect. 4), while the re-

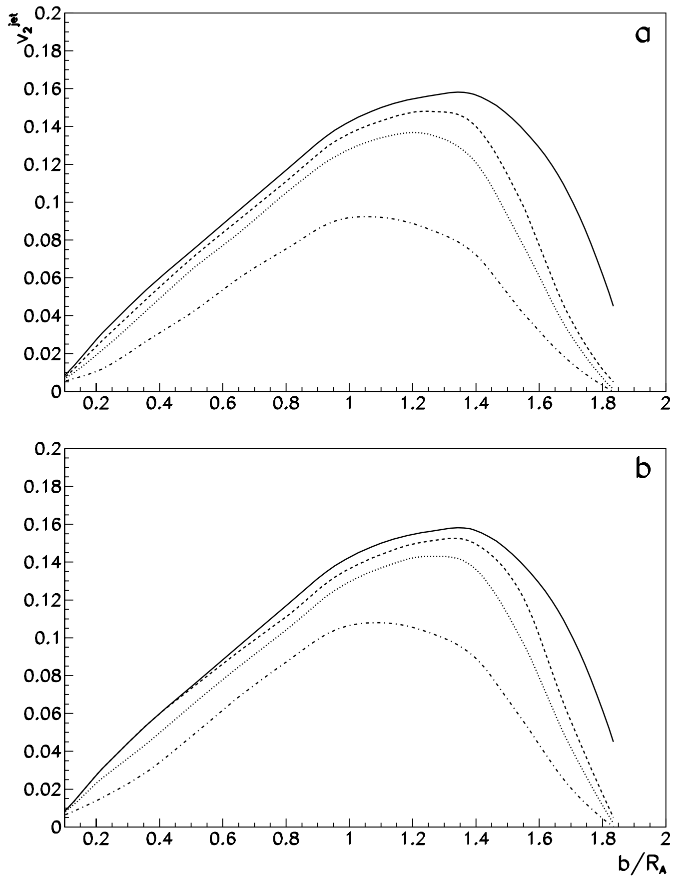

Fig. 1a,b. The impact parameter dependence of the "theoretical" value of v_2^{jet} including collisional and radiative energy losses (solid curve), and v_2^{jet} determined by the method (9) for $\bar{v}_{\mathrm{cor}}^{-}/\bar{v}_2^2 = 0$ (dashed curve), 0.01 (dotted curve) and 0.1 (dash-dotted curve). The result obtained using the fourth order cumulant method (15) coincides with the dashed curve. $\mathrm{d}N^{\pm}/\mathrm{d}y(y=0, b=0) = 3000$ **a** and 6000 **b**

sult of the calculation (9) changed, closely following the formula (26) corrected by autocorrelation terms which are non-vanishing in a finite summation [18].

We have also found at the calorimetric level that, taking into account the effect of a possible detector inefficiency (i.e. that the particles and jets are not detected in a "blind" azimuthal sector of size α), the accuracy of the v_2^{jet} determination appears to be less than 50% at $\alpha \gtrsim 30°$ in our model calculation without correlations and at $b \geq R_A$, whichever algorithm, (9) or (15), we used.

Figure 1 is presented to illustrate the improvement due to the fourth order cumulant method in the determination of jet azimuthal anisotropy v_2^{jet} depending on the ratio $\bar{v}_{\mathrm{cor}}^{-}/\bar{v}_2^2$, where $\bar{v}_2$ is the coefficient of the elliptic azimuthal anisotropy of the energy flow defined here as

$$\bar{v}_2 = \frac{1}{2}\,\frac{E_{\max(i)} - E_{\min(i)}}{E_{\max(i)} + E_{\min(i)}}, \tag{27}$$

and $E_{\max(i)}$ and $E_{\min(i)}$ are the maximum and minimum energy deposits in a segment respectively ($i = 1, ..., 72$). The coefficient $\bar{v}_{\mathrm{cor}}^{-}$ determines the "correlation strength" at the calorimetric level, $c_{ij} = 72\bar{v}_{\mathrm{cor}}^{-}\delta_{ij}$ for short-range correlations[5] (a similar result is obtained for long-range correlations with $c_{ij} = 2\bar{v}_{\mathrm{cor}}^{-}\cos 2(\varphi_i - \varphi_j)$). The plots show the b-dependence of the "theoretical" value of v_2^{jet} (calculated including collisional and radiative energy losses when the reaction plane angle is known in each event), and the v_2^{jet} determined by the methods (9) and (15) for the three ratios $\bar{v}_{\mathrm{cor}}^{-}/\bar{v}_2^2 = 0, 0.01, 0.1$. We used two values of the input parameter, the number of charged particles per unit rapidity at $y = 0$ in central Pb–Pb collisions: $\mathrm{d}N^{\pm}/\mathrm{d}y = 3000$ (Fig. 1a) and 6000 (Fig. 1b).

One can see that the improvement due to the fourth order cumulant method (the result of (15) is independent of $\bar{v}_{\mathrm{cor}}^{-}/\bar{v}_2^2$ and coincides with the result of (9) for $\bar{v}_{\mathrm{cor}}^{-}/\bar{v}_2^2 = 0$) is pronounced for more peripheral collisions, smaller particle multiplicities and larger "correlation strengths".

6 Conclusions

In the present paper we have analyzed the method for measurements of jet azimuthal anisotropy coefficients without reconstruction of the event plane considering the higher order correlators between the azimuthal position of the jet axis and the angles of particles not incorporated in the jet. The method is generalized by introducing as weights the particle momenta or the energy deposits in the calorimeter segments. In the latter case, we have illustrated its reliability in the real physical situation under LHC conditions. Introducing in the model correlations between energy deposits in the calorimeter segments does not practically change the accuracy of the method using fourth order cumulant calculations (15), while the result obtained with the second order correlator (9) is significantly dependent on the "strength" of such correlations. The advantage of the higher order cumulant analysis is pronounced for more peripheral collisions and smaller particle multiplicities.

To summarize, we believe that the present technique may be useful for investigating the azimuthal anisotropy of jets and high-p_{T} particles in heavy ion collisions at RHIC and LHC.

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[5] Here one should note that the majority of the sources of non-flow correlations mentioned above is effective at small angles between the particles. In our case these correlations can be partially smoothed out after summing particle energies over the azimuthal angles in a calorimeter segment of finite size ($\sim 5°$). This can result in a smaller value of the ratio $\bar{v}_{\mathrm{cor}}^{-}/\bar{v}_2^2$ in comparison with the ratio $v_{\mathrm{cor}}^{-}/v_2^2$ (and, as consequence, in a somewhat lower improvement due to the higher order method at the calorimetric level (15) in comparison with the particle level (14)). We still do not have an adequate Monte Carlo generator for particle flow effects at LHC including the physical model for correlations. Thus we cannot estimate the real value of $\bar{v}_{\mathrm{cor}}^{-}/\bar{v}_2^2$ (the true benefit of the higher order method) and just treat it here as a phenomenological parameter

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